Fourier Transformed Matrix Method of Finding Propagation Characteristics of Complex Anisotropic Layered Media

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Abstract —A planar structure having arbitrarily located conductor lines immersed in complex anisotropic media presents one with a very general guided wave problem. This problem is solved here by a rigorous formulation technique characterizing each layer by a 6×6 tensor and finding the appropriate Fourier transformed Green's function matrix G of $2n\times 2n$ size. From G, a method-of-moments solution for the propagation characteristics follows, including propagation constant eigenvalues and field eigenvectors at all spatial locations. The method is very versatile and can handle a large class of microwave or millimeter-wave integrated circuit or monolithic circuit problems, no matter how simple or complex as long as they possess planar symmetry.

I. Introduction

ADVANCES in materials technology are allowing the contiguous growth of substances of considerably different properties. Present integrated-circuit processing techniques allow various combinations of metals, dielectrics, and semiconductors to be layered together where these materials may or may not be crystalline. Processing can include vapor phase (VPE), liquid phase (LPE), or molecular beam expitaxial (MBE) growth for deposition of layers of semiconductors or dielectrics. An additional method for metal deposition includes electron-beam heating and vacuum deposition. The number of processing techniques are numerous and have varying degrees of success in enabling dissimilar materials to be deposited adjacently. These techniques display a large range of capabilities to make defect-free layers, uniform doping level semiconductors, and controllable layer thicknesses.

We may expect to see in the future the use of magnetic films [1], [2] (metallic or nonmetallic), uniaxial and biaxial dielectric films [3]–[6], ferrite films, magnetically induced semiconductor gyroelectric films, and widely varying compositions of compound films such as binary, ternary, and quaternary compounds [7]. MBE and some other methods are encouraging the use of layers which are thin enough (on the order of tens to hundreds of angstroms) to require modification of the three-dimensional (3D) transport analysis (semiclassical or quantum mechanical) so as to include the two-dimensional (2D) electron gas effect. The change from 3D to 2D transport on the microscopic level after appropriate analysis can be accounted for in terms of macroscopic permittivity and permeability tensors.

Manuscript received March 23, 1984; revised July 10, 1984.
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More creative use of materials, especially for monolithic integrated circuits, will probably occur in the future. Besides some of the more familiar classes of anisotropic materials mentioned above, materials with optical activity may be employed in integrated-circuit applications. Finally, very complex materials with a combination of birefringent gyroelectric, gyromagnetic, optical rotation, or other anisotropic properties may be utilized. Furthermore, the use of semiconductors, dielectrics, or magneto-materials rotated off principal axes or convenient axis coordinates can be envisioned.

Conventional methods either in direct or Fourier-transformed space using planar symmetry are not general enough to enable the interested worker in the microwave or millimeter-wave area to readily solve such complex problems outlined above. The theoretical formulation presented in this paper shows how to solve for the propagation constant γ and electromagnetic fields in a multilayered planar structure having complex anisotropic layers. From the field and γ solution, the characteristic impedance Z_c may also be determined by an appropriate definition in terms of a current–voltage power flow combination. The only restrictions to the formulation below are that the conductors are assumed to be lossless and infinitesimally thick.

Expeditious ways of solving field problems based on Maxwell's equations, but avoiding gauge methods, are possible by using field matrix techniques. Solving the field problem by eliminating all but one field component variable generates a fourth-order partial differential equation (PDE) for linear media. The resulting fourth-order PDE can be extremely complex and unwieldy to handle; however, there is some justification for developing the solution in such a manner if the medium has special symmetry. Simpler PDE's result if the problem is reduced down to two field components. Two second-order PDE's must be employed in two field components to find the field solution. Matrix techniques using two field components often have been used in the optics [8], [9] and microwave/electromagnetics [10] areas. Nevertheless, as the medium becomes more complex with less symmetry, the two-component methods become increasingly difficult to implement. Lack of conductor line symmetry also complicates the two-component solution methods.

Use of four components has been shown in reflection and transmission light problems to lead to simple PDE's

0018-9480/84/1200-1617$01.00

[11]; [12]. A four-component method has the great advantage of enabling the use of only first-order PDE's. The four-component technique also has the ability to allow direct field matching at layer boundaries or interfaces [13]. In [13], it is pointed out that such field matching avoids the need to employ auxiliary equations in the two suppressed field components if using the two-component method, thus providing some economy in problem solving.

Here, a new formulation technique for solving the uniform (in the z-direction) waveguide propagation problem is developed for layered media possessing complex anisotropic properties. A four-component method is utilized by adapting the 4×4 matrix approach in [13] to the spectral domain or Fourier-transform domain (FTD). Significant advantage is gained by working in the FTD because Green's function convolution integrals for determination of field quantities due to current sources are converted into algebraic products. The FTD process, in addition, treats asymmetrical conductor lines in the same way as it treats symmetrical conductor lines. Section II develops the normal mode field solution formulation in the FTD. One should be alerted that the chosen column field vector used here differs from [13] in the component selection and arrangement. Section III gives the solution to the open or uncovered multilayered guided-wave problem. The Green's function G which is derived pertains to a perfectly conducting ground plane. From G, the procedure for determining the propagation constant γ is provided for the method-of-moments numerical technique assisted by identical expansion and test basis functions (Galerkin approach). Determination of electromagnetic field components using γ is also covered. Section IV provides the solution to the closed guided-wave problem where the electromagnetic fields are sandwiched between two ground planes.

II. Normal Mode Field Solution

Each layer has four eigenfunction field solution sets. Superposition of these four normal mode sets of field components constitutes the actual total field solution obeying all boundary conditions (BC). In this section, the normal modes are found for the mth layer without regard to the BC's. Imposition of the various BC's as part of the overall propagation constant γ determination is done in the following sections, where specific waveguide structures are considered.

Time harmonic, plane guided-wave solutions proportional to $\exp(j\omega t-\gamma z)$ are assumed. Propagation constant $\gamma=\alpha+j\beta$ makes the wave $+z$-direction propagating if $\beta>0$. Insertion of the time harmonic nature of the plane wave into Maxwell's two curl equations creates the single sourceless matrix equation

$$L_T V_L' = j\omega V_R'. \tag{1}$$

Magnetic current sources are not considered in the treatment in this paper and electric current sources are included by discontinuity BC's at interfaces between layers. V_L' and V_R' are column vectors containing electromagnetic-field components in rectangular coordinates both tangential to the parallel interfaces (xz-plane) and parallel to the y-axis. V_L' consists of the electric-field $\boldsymbol{E}$ and magnetic-field $\boldsymbol{H}$ components. V_R' consists of the electric displacement field $\boldsymbol{D}$ and the magnetic displacement field $\boldsymbol{B}$ components

$$V_L'=\begin{bmatrix} E_x' \\ E_y' \\ E_z' \\ H_x' \\ H_y' \\ H_z' \end{bmatrix}, \qquad V_R'=\begin{bmatrix} D_x' \\ D_y' \\ D_z' \\ B_x' \\ B_y' \\ B_z' \end{bmatrix}. \tag{2}$$

Primes indicate that these vectors contain x, y, and z spatial variation with the z spatial variation to be dropped shortly. Operator L_T is a 6×6 matrix composed of single partial-derivative operators. L_T can be expressed as

$$L_T=\left[\begin{array}{c|c} 0 & L_1 \\ \hline -L_1 & 0 \end{array}\right] \tag{3}$$

where submatrix L_1 is a 3×3 matrix

$$L_1=\begin{bmatrix} 0 & -\dfrac{\partial}{\partial z} & \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} & 0 & -\dfrac{\partial}{\partial x} \\ -\dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial x} & 0 \end{bmatrix}. \tag{4}$$

To remove explicit z spatial dependence from (1), V_L and V_R are defined as

$$V_L = V_L' e^{\gamma z} \tag{5a}$$

$$V_R = V_R' e^{\gamma z}. \tag{5b}$$

The uniform (in z-direction) guided-wave problem with (5) can now be solved in transformed space by going from x direct space to k_x reciprocal space. One-dimensional Fourier-transform pair $(f,\tilde{f})$ is defined as

$$\tilde{f}(k_x,y)=\int_{-\infty}^{\infty} f(x,y)\,e^{-jk_x x}\,dx \tag{6a}$$

$$f(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{f}(k_x,y)\,e^{jk_x x}\,dk_x \tag{6b}$$

where $f(x,y)$ is any real space variable. Using (5) and (6) allows conversion of the guided-wave problem as given by (1) into the FTD

$$\tilde{L}_T \tilde{V}_L = j\omega \tilde{V}_R \tag{7}$$

where tildes denote FTD variables and

$$\tilde{L}_T=\left[\begin{array}{c|c} 0 & \tilde{L}_1 \\ \hline -\tilde{L}_1 & 0 \end{array}\right] \tag{8}$$

$$\tilde{L}_1=\begin{bmatrix} 0 & \gamma & \dfrac{d}{dy} \\ -\gamma & 0 & -jk_x \\ -\dfrac{d}{dy} & jk_x & 0 \end{bmatrix}. \tag{9}$$

Fourier-transformed electromagnetic-fields $\tilde{V}_L$ and $\tilde{V}_R$ are

$$\tilde{V}_L = \begin{bmatrix} \tilde{E}_x \\ \tilde{E}_y \\ \tilde{E}_z \\ \tilde{H}_x \\ \tilde{H}_y \\ \tilde{H}_z \end{bmatrix}, \quad \tilde{V}_R = \begin{bmatrix} \tilde{D}_x \\ \tilde{D}_y \\ \tilde{D}_z \\ \tilde{B}_x \\ \tilde{B}_y \\ \tilde{B}_z \end{bmatrix}. \tag{10}$$

The medium of each layer can be characterized by a single 6×6 constitutive tensor $\hat{M}$ in direct space. $\hat{M}$ relates V_R' to V_L'

$$V_R' = \hat{M} V_L' \tag{11}$$

where

$$\hat{M} = \left[\begin{array}{c|c} \hat{\epsilon} & \hat{\rho} \\ \hline \hat{\rho}' & \hat{\mu} \end{array}\right]. \tag{12}$$

$\hat{\epsilon}$ and $\hat{\mu}$ are, respectively, the permittivity and permeability tensors. $\hat{\epsilon}$ can lead to electric birefringence in uniaxial and biaxial dielectric crystals. Gyroelectric nonreciprocal behavior can be induced through $\hat{\epsilon}$. This may occur by applying a magnetic field to a doped semiconductor layer in a Faraday, Voigt, or mixed configuration. Similarly, using $\hat{\mu}$, magnetic birefringence can be obtained in a dual fashion in uniaxial and biaxial magnetic crystals. Gyromagnetic nonreciprocal properties can be had by applying a magnetic field to a ferrite material, for example. $\hat{\rho}$ and $\hat{\rho}'$ tensors are responsible for optical activity. $\hat{M}$ could create nonlinear effects by being dependent on the field vector V_L'. $\hat{M}$ also could be dependent on the coordinates x and y. It is possible that $\hat{M}$ might be dependent on both the field vector and coordinates. In order to maintain a linear problem solution, $\hat{M}$ is treated as a constant tensor. x-coordinate variation of $\hat{M}$ cannot be built back into the problem solution by such an assumption, but y-coordinate variation can be by slicing the layer with $\hat{M}$ y-dependence into thin sections. Fourier transforming (11) and using (5) yields

$$\tilde{V}_R = \hat{M} \tilde{V}_L. \tag{13}$$

Placing (13) into (7) produces the equation which must be solved for the normal mode field vectors

$$\tilde{L}_T \tilde{V}_L = j\omega \hat{M} \tilde{V}_L. \tag{14}$$

$\tilde{V}_2$ and $\tilde{V}_5$ vector components of $\tilde{V}_L$ (i.e., $\tilde{E}_y, \tilde{H}_y$) are algebraically expressible in terms of the other field components using rows 2 and 5 of (14)

$$-\gamma \tilde{V}_4 - jk_x \tilde{V}_6 = j\omega \sum_{i=1}^{6} m_{2i} \tilde{V}_i \tag{15a}$$

$$\gamma \tilde{V}_1 + jk_x \tilde{V}_3 = j\omega \sum_{i=1}^{6} m_{5i} \tilde{V}_i. \tag{15b}$$

The solution of (15) is

$$\tilde{V}_i = \sum_{j=1}^{6} a_{ij} (1 - \delta_{2,j})(1 - \delta_{5,j}) \tilde{V}_j, \qquad i = 2,5 \tag{16}$$

where $\delta_{i,j}$ are Kronecker deltas. a_{ij} coefficients are

$$a_{ij} = \frac{a_{ij}'}{D_a} \tag{17}$$

$$D_a = m_{22} m_{55} - m_{25} m_{52} \tag{18}$$

$$a_{21}' = m_{25}\left(m_{51} - \frac{\gamma}{j\omega}\right) - m_{21} m_{55} \tag{19a}$$

$$a_{23}' = m_{25}\left(m_{53} - \frac{k_x}{\omega}\right) - m_{23} m_{55} \tag{19b}$$

$$a_{24}' = m_{25} m_{54} - m_{55}\left(m_{24} + \frac{\gamma}{j\omega}\right) \tag{19c}$$

$$a_{26}' = m_{25} m_{56} - m_{55}\left(m_{26} + \frac{k_x}{\omega}\right) \tag{19d}$$

$$a_{51}' = m_{52} m_{21} - m_{22}\left(m_{51} - \frac{\gamma}{j\omega}\right) \tag{20a}$$

$$a_{53}' = m_{52} m_{23} - m_{22}\left(m_{53} - \frac{k_x}{\omega}\right) \tag{20b}$$

$$a_{54}' = m_{52}\left(m_{24} + \frac{\gamma}{j\omega}\right) - m_{22} m_{54} \tag{20c}$$

$$a_{56}' = m_{52}\left(m_{26} + \frac{k_x}{\omega}\right) - m_{22} m_{56}. \tag{20d}$$

In (18)–(20), m_{ij} are the $\hat{M}$ tensor elements.

Rows 1, 3, 4, and 6 of (14) are first-order linear differential equations

$$\gamma \tilde{V}_5 + \frac{d\tilde{V}_6}{dy} = j\omega \sum_{i=1}^{6} m_{1i} \tilde{V}_i \tag{21a}$$

$$-\frac{d\tilde{V}_4}{dy} + jk_x \tilde{V}_5 = j\omega \sum_{i=1}^{6} m_{3i} \tilde{V}_i \tag{21b}$$

$$-\gamma \tilde{V}_2 - \frac{d\tilde{V}_3}{dy} = j\omega \sum_{i=1}^{6} m_{4i} \tilde{V}_i \tag{21c}$$

$$\frac{d\tilde{V}_1}{dy} - jk_x \tilde{V}_2 = j\omega \sum_{i=1}^{6} m_{6i} \tilde{V}_i. \tag{21d}$$

Using (16) to eliminate $\tilde{V}_2$ and $\tilde{V}_5$ from (21) produces

$$\frac{1}{j\omega}\frac{d}{dy}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_3 \\ \tilde{V}_4 \\ \tilde{V}_6 \end{bmatrix} = R' \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_3 \\ \tilde{V}_4 \\ \tilde{V}_6 \end{bmatrix}. \tag{22}$$

In (22), the matrix elements r_{ij}' of R' are

$$r_{1i}' = m_{6\theta} + a_{5\theta} m_{65} + a_{2\theta}\left(m_{62} + \frac{k_x}{\omega}\right) \tag{23a}$$

$$r_{2i}' = m_{4\theta} + a_{5\theta} m_{45} + a_{2\theta}\left(m_{42} + \frac{\gamma}{j\omega}\right) \tag{23b}$$

$$r_{3i}' = m_{3\theta} + a_{2\theta} m_{32} + a_{5\theta}\left(m_{35} - \frac{k_x}{\omega}\right) \tag{23c}$$

$$r_{4i}' = m_{1\theta} + a_{2\theta} m_{12} + a_{5\theta}\left(m_{15} - \frac{\gamma}{j\omega}\right) \tag{23d}$$

$$\theta=\theta(i)=\begin{cases}\dfrac{3}{2}i, & i=\text{even}\\[2mm] \dfrac{3i-1}{2}, & i=\text{odd}\end{cases},\qquad i=1,2,3,4. \tag{24}$$

Equation (22) reduces to a more convenient form when both sides of it are multiplied by the prefactor matrix S_p found on the left-hand side. Since $S_p \cdot S_p = S_p^2 = I$, the identity 4×4 matrix, (22) becomes

$$\frac{1}{j\omega}\frac{d\phi}{dy}=R\phi. \tag{25}$$

ϕ is the four-element column vector in the FTD having only interface tangential field components

$$\phi=\begin{bmatrix}\tilde{V}_1\\ \tilde{V}_3\\ \tilde{V}_4\\ \tilde{V}_6\end{bmatrix}=\begin{bmatrix}\tilde{E}_x\\ \tilde{E}_z\\ \tilde{H}_x\\ \tilde{H}_z\end{bmatrix}. \tag{26}$$

R is composed of matrix elements r_{ij} related to the R' matrix elements by

$$\left.\begin{aligned} r_{ij}&=r'_{ij}, & i&=1,4\\ r_{ij}&=-r'_{ij}, & i&=2,3\end{aligned}\right\},\qquad j=1,2,3,4. \tag{27}$$

Translate y into the mth local layer shifted coordinate system

$$y'_m=y-\sum_{j=1}^{m-1}h_j \tag{28}$$

where h_j is the jth layer thickness and $y'_m=0$ corresponds to an interface. Solutions to (25) in the y'_m coordinate system can be written as

$$\phi(y'_m)=e^{jk_y y'_m}\phi(0). \tag{29}$$

Substituting (29) into (25) produces

$$\left\{\frac{k_{yi}}{\omega}I-R\right\}\phi_i(0)=0 \tag{30}$$

where the attached i subscript's meaning will be clarified shortly. Equation (30) is a homogeneous equation in four unknown ϕ vector components. It has four normal mode vector solutions $\phi_i(0)$, constrained by the requirement that

$$\det\left(\frac{k_{yi}}{\omega}I-R\right)=0 \tag{31}$$

to assure a solution. Equation (31) generates four k_y eigenvalues k_{yi}. These k_{yi} values are placed in (30) to find the individual $\phi_i(0)$ normal mode vectors at $y'_m=0$. The normal mode vector $\phi_i^m(y'_m)$ at $y'_m>0$ is found from $\phi_i^m(0)$ by multiplication with a 4×4 matrix characterizing the mth layer medium

$$\phi_i^m(y'_m)=P^m(y'_m)\phi_i^m(0). \tag{32}$$

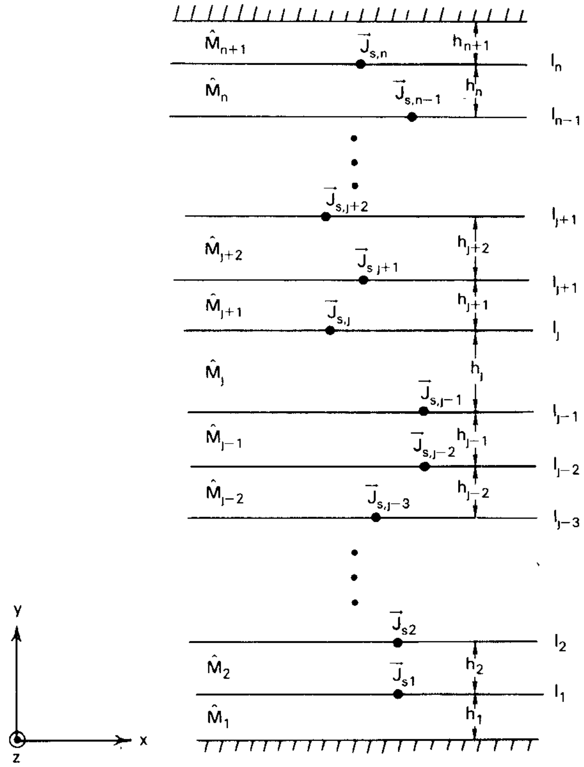

Fig. 1. Cross section of guided-wave structure containing $n+1$ layers of thickness h_j, each characterized by a 6×6 constitutive tensor $\hat{M}_j$. There are n interface impulse surface current line sources $J_{s,j}$. The whole structure is bounded by two perfectly conducting electric walls.

Here

$$P^m(y'_m)=\Psi^m(0)K^m(y'_m)\Psi^m(0)^{-1} \tag{33}$$

$$K_{ij}^m=\delta_{ij}\exp\left(jk_{yi}^m y'_m\right) \tag{34}$$

$$\Psi^m(0)=\left[\phi_1^m(0)\quad \phi_2^m(0)\quad \phi_3^m(0)\quad \phi_4^m(0)\right]. \tag{35}$$

The superscript m in (32)–(35) emphasizes the specialization of the eigenvector solution to the mth layer. $\Psi^m(0)$ is a 4×4 matrix constructed out of the four normal mode vectors. $\phi_i^m(y'_m)$ given by (32) is used in succeeding sections to construct Green's functions for waveguiding structures of concern and to proceed on to a solution for the propagation constant γ.

III. Half-Open Guided-Wave Structure

Fig. 1 shows the structure of the waveguiding layered configuration. It has top and bottom perfectly conducting ground planes. Layers are characterized by thicknesses h_j, there being a total of $n+1$ layers. There are n interfaces between the layers, and a total of n interface surface currents J_s^j. These surface currents are line sources and may be thought of in an abstract sense as representing perfectly conducting lines positioned at the interfaces. Consequently, we set the jth surface current as

$$J_s^j=(\hat{x}+\hat{z})\delta(x-x_j). \tag{36}$$

J_s^j in (36) is a vector impulse line source located at $x = x_j$ and at the jth interface with $y_j = \Sigma_{m=1}^{j-1} h_m$ [see (28)]. Solution of the guided-wave problem using (36) produces a Green's function field solution for the electric-field components. Below in the FTD derivation of the electric-field solution, J_s^j components are retained avoiding explicit use of the spatial form in (36). This procedure is followed because Green's function convolution integrals over actual current distributions on conductors in the spatial domain become a product of a Green's function dyadic G and FTD currents. Implementation of the procedure involves replacing the FTD expressions of J_s^j in (36) by the FTD expressions of the actual current distributions. Each layer is characterized by a single constitutive tensor $\hat{M}_j$ (see (12)) for the jth layer, there being a total of $n+1$ tensors for all layers. The open structure to be analyzed is mathematically and physically created by letting $h_{n+1} \to \infty$. In the $(n+1)$th layer, only $+y$ outward propagating (or decaying) waves are desired in this limit, and this fact is utilized later to find the Green's function.

The normal mode or eigenvector solutions found in Section II must be superpositioned to represent the actual (total) field solution in each layer $\Lambda^m(y_m')$

$$\Lambda^m(y_m') = \sum_{i=1}^{4} e_i \phi_i^m(y_m'). \tag{37}$$

The mapping of $\Lambda^m(0)$ into $\Lambda^m(y_m')$, which is essential to the derivation below, is readily found using (32)

$$\Lambda^m(y_m') = \sum_{i=1}^{4} e_i P^m(y_m') \phi_i^m(0) = P^m(y_m') \sum_{i=1}^{4} e_i \phi_i^m(0) = P^m(y_m') \Lambda^m(0). \tag{38}$$

The four-element column vector Λ^m is convenient to use for applying interface or boundary conditions, since the BC's can be expressed solely in terms of tangential field components. Electric-field components Λ_1^m and Λ_2^m ($\tilde{E}_x^m$ and $\tilde{E}_z^m$) at the mth interface are continuous. Magnetic-field components Λ_3^m and Λ_4^m ($\tilde{H}_x^m$ and $\tilde{H}_z^m$) are discontinuous

$$\tilde{H}_x(h_m^+) - \tilde{H}_x(h_m^-) = -\tilde{J}_{sz}^m \tag{39a}$$

$$\tilde{H}_z(h_m^+) - \tilde{H}_z(h_m^-) = \tilde{J}_{sx}^m \tag{39b}$$

as stated in the local mth layer coordinate system where "+" or "−" denotes infinitesimal displacement along the y-axis (h_m^+ actually goes into the $[m+1]$th local system). The BC's at the interfaces in matrix notation are

$$\begin{bmatrix} \tilde{E}_x^m \\ \tilde{E}_z^m \\ \tilde{H}_x^m \\ \tilde{H}_z^m \end{bmatrix} (h_m^+) = \begin{bmatrix} \tilde{E}_x^m \\ \tilde{E}_z^m \\ \tilde{H}_x^m \\ \tilde{H}_z^m \end{bmatrix} (h_m^-) + \begin{bmatrix} 0 \\ 0 \\ -\tilde{J}_{sz}^m \\ \tilde{J}_{sx}^m \end{bmatrix}$$

or in abbreviated form

$$\Lambda^m(h_m^+) = \Lambda^m(h_m^-) + \begin{bmatrix} 0 & 0 & -\tilde{J}_{sz}^m & \tilde{J}_{sx}^m \end{bmatrix}^T \tag{40}$$

where the superscript T on the last vector means transpose.

$\Lambda^m(y_m')$ in the mth layer is found by starting at $y_1' = 0$ and proceeding through each layer using (38), accounting for field match or mismatch along the way at successive interfaces by enlisting (40). The result of this procedure is

$$\Lambda^m(y_m') = F^{m,m} \Lambda^1(0) + \sum_{k=1}^{m-1} F^{m,m-k} \begin{bmatrix} 0 & 0 & -\tilde{J}_{sz}^k & \tilde{J}_{sx}^k \end{bmatrix}^T \tag{41}$$

$$F^{m,l}(y_m') = \prod_{j=m-l+1}^{m} P^j(h_j'). \tag{42}$$

In (42), $h_j' = h_j$, the jth layer thickness, except for the mth layer where $h_j' = y_m'$. Equation (42) creates left ascending products. $F^{m,l}$ in (42) has a simple but elegant physical interpretation. It is a mapping prefactor operator which takes the quantity to its right and pulls it through l layers until it arrives at y_m' in the mth layer. Applied to (41), the operator with $l = m$ takes the field vector $\Lambda^1(0)$ and draws it through all m layers to the position y_m' in the final mth layer. When $l = m - k$ as in the second operator acting on the kth discontinuity BC, the operator pulls the BC sitting on the kth interface, which is on top of the kth layer and on the bottom of the $k+1$ layer, through the remaining $m-k$ layers.

$\Lambda^1(0)$ is unknown in (41) and needs to be determined so that $\Lambda^m(y_m')$ is uniquely specified. The first two components of $\Lambda^1(0)$ are $\Lambda_1^1(0) = \Lambda_2^1(0) = 0$ due to the ground plane BC. $\Lambda_3^1(0)$ and $\Lambda_4^1(0)$ are determined by using (41) to connect the ground plane at $y = y_1' = 0$ and the $(n+1)$th side of the nth interface. Invoking (41)

$$\Lambda^{n+1}(0) = F^{n+1,n+1}(0)\Lambda^1(0) + \sum_{k=1}^{n} F^{n+1,n+1-k}(0) \begin{bmatrix} 0 & 0 & -\tilde{J}_{sz}^k & \tilde{J}_{sx}^k \end{bmatrix}^T. \tag{43}$$

Here, one notes that $F^{n+1,n+1-k}(0) = F^{n,n-k}(h_n)$ for $n \geqslant k \geqslant 0$. $\Lambda^{n+1}(0)$ must be specified in order to solve (43) for the necessary $\Lambda^1(0)$ components. First the normal modes comprising $\Lambda^{n+1}(0)$ need to be obtained. This layer is isotropic with [see (12)]

$$\hat{M}_{n+1} = \left[\begin{array}{c|c} \epsilon_{n+1} I & 0 \\ \hline 0 & \mu_{n+1} I \end{array}\right]. \tag{44}$$

a_{ij} is available from (17), with $D_a = \epsilon_{n+1}\mu_{n+1}$ by (44). Defining a_2' and a_5' as the vectors containing their respective a_{ij}', $i = 2$ or 5, $j = 1, 3, 4$ and 6

$$a_2' = \begin{bmatrix} 0 \\ 0 \\ \dfrac{j\gamma\mu_{n+1}}{\omega} \\ -\dfrac{k_x\mu_{n+1}}{\omega} \end{bmatrix}, \quad a_5' = \begin{bmatrix} -\dfrac{j\gamma\epsilon_{n+1}}{\omega} \\ \dfrac{k_x\epsilon_{n+1}}{\omega} \\ 0 \\ 0 \end{bmatrix}. \tag{45}$$

Inserting (44) and (45) into (23), using (17) and (24), allows R' to be established

$$R' = \begin{bmatrix} 0 & 0 & \dfrac{j\gamma k_x}{\omega^2\epsilon_{n+1}} & \left(\mu_{n+1} - \dfrac{k_x^2}{\omega^2\epsilon_{n+1}}\right) \\ 0 & 0 & \left(\mu_{n+1} + \dfrac{\gamma^2}{\omega^2\epsilon_{n+1}}\right) & \dfrac{j\gamma k_x}{\omega^2\epsilon_{n+1}} \\ \dfrac{j\gamma k_x}{\omega^2\mu_{n+1}} & \left(\epsilon_{n+1} - \dfrac{k_x^2}{\omega^2\mu_{n+1}}\right) & 0 & 0 \\ \left(\epsilon_{n+1} + \dfrac{\gamma^2}{\omega^2\mu_{n+1}}\right) & \dfrac{j\gamma k_x}{\omega^2\mu_{n+1}} & 0 & 0 \end{bmatrix}. \tag{46}$$

Normal mode eigenvalues k_{yi} are found by placing (46) into (31) assisted by (27). The dispersion equation

$$\left\{k_{yi}^2 - \left[k_{n+1}^2 - k_x^2 + \gamma^2\right]\right\}^2 = 0 \tag{47}$$

results with $k_{n+1}^2 = \omega^2\epsilon_{n+1}\mu_{n+1}$. Equation (47) produces two distinct eigenvalues, or a total of four normal modes, two being degenerate. Placing k_{yi} into (30) and using three of the four implied equations produces the eigenvectors $\phi_i^{n+1}(0)$. We choose k_{yi}, $i = 1$ or 2, to be the distinct eigenvalues, and denote the two different eigenvectors associated with each i by subscripts a and b. Because the $(n+1)$th layer is semi-infinite, only the outward propagating (or decaying) wave in the $+y$ direction is displayed below:

$$\phi_{1a}^{n+1}(0) = \begin{bmatrix} \dfrac{\omega\mu_{n+1}k_{y1}^{n+1}}{k_{n+1}^2+\gamma^2} \\ 0 \\ -\dfrac{j\gamma k_x}{k_{n+1}^2+\gamma^2} \\ 1 \end{bmatrix}, \quad \phi_{1b}^{n+1}(0) = \begin{bmatrix} -\dfrac{j\gamma k_x}{k_{n+1}^2+\gamma^2} \\ 1 \\ -\dfrac{\omega\epsilon_{n+1}k_{y1}^{n+1}}{k_{n+1}^2+\gamma^2} \\ 0 \end{bmatrix}. \tag{48}$$

Notice that this degenerate normal mode solution consists of TM_z and TE_z forms. Since the medium is isotropic, the solution could have been resolved into other TM_n and TE_n forms where n = rectangular axis.

Equations (48) are used to construct the total field vector at $y'_{n+1} = 0$

$$\Lambda^{n+1}(0) = A_{n+1}\phi_{1a}^{n+1}(0) + B_{n+1}\phi_{1b}^{n+1}(0). \tag{49}$$

Equating (49) to (43) creates a single vector equation comprised of four linear algebraic equations in four unknowns A_{n+1}, B_{n+1}, $\Lambda_3^1(0)$, and $\Lambda_4^1(0)$. $\Lambda_3^1(0)$ and $\Lambda_4^1(0)$ are solved as

$$\Lambda_j^1(0) = \sum_{i=1}^{4} C_i b_{ji};\ j = 3,4\ (or\ x, z) \tag{50}$$

$$C_i = \sum_{m=1}^{n-1}\left(-F_{i3}^{n,n-m}\tilde{J}_{sz}^m + F_{i4}^{n,n-m}\tilde{J}_{sx}^m\right) - \delta_{i3}\tilde{J}_{sz}^n + \delta_{i4}\tilde{J}_{sx}^n \tag{51}$$

$$b_{ji} = -(-1)^j\frac{1}{D_o}\det\left[\bar{\phi}_{\bar{i}a} \quad \bar{\phi}_{\bar{i}b} \quad -\bar{F}_{\bar{i},7-j}\right] \tag{52}$$

$$D_o = \det\left[\phi_{1a} \quad \phi_{1b} \quad -\bar{F}_3 \quad -\bar{F}_4\right] \tag{53}$$

$$\bar{\phi}_{\bar{i}(a,b)} = \begin{bmatrix} \phi_{1j(a,b)}^{n+1} \\ \phi_{1k(a,b)}^{n+1} \\ \phi_{1l(a,b)}^{n+1} \end{bmatrix}(0) \qquad \bar{F}_{\bar{i},(3,4)} = \begin{bmatrix} F_{j(3,4)}^{n,n} \\ F_{k(3,4)}^{n,n} \\ F_{l(3,4)}^{n,n} \end{bmatrix} \tag{54a,b}$$

$$\bar{F}_{(3,4)} = \begin{bmatrix} F_{1(3,4)}^{n,n} \\ F_{2(3,4)}^{n,n} \\ F_{3(3,4)}^{n,n} \\ F_{4(3,4)}^{n,n} \end{bmatrix}. \tag{55}$$

In (54), the component subscripts j, k, l are cyclic, exclude i (notation $\bar{i}$), and equal 1 through 4. Equation (50) is inserted into (41) so that $\Lambda^m(y'_m)$ is determined. The $\tilde{E}_x^m(h_m)$ and $\tilde{E}_z(h_m)$ electric-field components are then extracted out of the resulting (41) and related to all the interface surface current vectors (or surface current components). The identical procedure is carried out for each interface set of electric-field components, yielding a total of n interface electric-field component sets related to n interface surface current vectors. Dyadic G relates these two sets.

The Green's function G in the FTD relating the field components at each interface U to all the interface currents W, $U = GW$, is explicitly defined by

$$U = \begin{bmatrix} \tilde{E}_1^1 \\ \tilde{E}_2^1 \\ \vdots \\ \tilde{E}_1^n \\ \tilde{E}_2^n \end{bmatrix} = \begin{bmatrix} S^{11} & S^{12} & \cdots & S^{1n} \\ \vdots & & & \vdots \\ S^{n1} & S^{n2} & \cdots & S^{nn} \end{bmatrix} \begin{bmatrix} \tilde{J}_{s1}^1 \\ \tilde{J}_{s2}^1 \\ \vdots \\ \tilde{J}_{s1}^n \\ \tilde{J}_{s2}^n \end{bmatrix} = GW. \tag{56}$$

U and W are electric-field component and interface current component vectors of size $2n$. Subscript indices on their field or current elements above denote $x(j=1)$ or $z(j=2)$ components. Elements of U are u_i, and of W, w_i. G is a $2n \times 2n$ matrix with each 2×2 S^{mp} submatrix element given by

$$S_{ij}^{mp} = T_{ij}^{mp} + F_{i,5-j}^{m,m-p} \tag{57}$$

$$T_{ij}^{mp} = \begin{cases} -(-1)^j \sum_{k=1}^{4} F_{k,5-j}^{n,n-p}(b_{xk}F_{i3}^{m,m} + b_{zk}F_{i4}^{m,m}), & p=1,2,\cdots,n-1 \quad (58a) \\ -(-1)^j(b_{x,5-j}F_{i3}^{m,m} + b_{z,5-j}F_{i4}^{m,m}), & p=n. \quad (58b) \end{cases}$$

In (57), the second term is dropped if $p \geqslant m$.

Propagation constant γ is obtained from (56), the starting point of a Galerkin process, by using basis (expansion) and test functions. Represent each element of the W vector as

$$w_i(k_x, \gamma) = \sum_{s=1}^{N_i} q_{is}(\gamma) w_{is}(k_x, \gamma) \tag{59}$$

where w_{is} are the FTD basis functions which may be chosen to be a complete set and q_{is} are weight coefficients. Multiply rows 1 through $2n$ of (56) by, respectively, $w_{1s'}^*$ through $w_{(2n)s'}^*$ with $s'=1,2,\cdots,N_i$ creating N_i equations for the ith row. The total number of equations will be $N=\sum_{i=1}^{2n} N_i$. Integrate these equations over reciprocal k_x space (drop the $1/2\pi$ factor) obtaining

$$\begin{aligned} Y_{1s'} &= \sum_{s=1}^{N_1} X_{s's}^{11} q_{1s} + \cdots + \sum_{s=1}^{N_{2n}} X_{s's}^{1(2n)} q_{(2n)s}, \qquad s'=1,2,\cdots,N_1 \\ &\vdots \\ Y_{(2n)s'} &= \sum_{s=1}^{N_1} X_{s's}^{(2n)1} q_{1s} + \cdots + \sum_{s=1}^{N_{2n}} X_{s's}^{(2n)(2n)} q_{(2n)s}, \qquad s'=1,2,\cdots,N_{2n} \end{aligned} \tag{60}$$

where

$$X_{s's}^{ij} = \int_{-\infty}^{\infty} w_{is'}^* G_{ij} w_{js}\, dk_x \tag{61}$$

$$Y_{is'} = \int_{-\infty}^{\infty} u_i w_{is'}^*\, dk_x. \tag{62}$$

Equations (60) represent N equations in N unknown q_{is} so that one can write

$$Y = S_x Q \tag{63}$$

with

$$S_x = \left[\begin{array}{c|c|c|c} X^{11} & X^{12} & \cdots & X^{1(2n)} \\ \hline \vdots & \vdots & & \vdots \\ \hline X^{(2n)1} & X^{(2n)2} & \cdots & X^{(2n)(2n)} \end{array}\right] \tag{64}$$

$$Q = [q_{11} \quad q_{12} \quad \cdots \quad q_{1N_1} \quad q_{21} \quad \cdots \quad q_{(2n)N_{2n}}]^T. \tag{65}$$

Each $X_{s's}^{ij}$ is the $s's$th element of the X^{ij} submatrix S_x. Using FTD properties

$$\begin{aligned} Y_{is'}(\gamma) &= \int_{-\infty}^{\infty} u_i(k_x,\gamma) w_{is'}^*(k_x,\gamma)\, dk_x \\ &= 2\pi \int_{-\infty}^{\infty} u_i(x,\gamma) w_{is'}^*(x,\gamma)\, dx. \end{aligned} \tag{66}$$

$u_i(x,\gamma)$ and $w_{is'}(x,\gamma)$ are the electric field and current distributions on the interfaces. If one assumes perfect conductors on the interfaces, $w_{is'}=0$ when $u_i \neq 0$ and the converse. The complementary nature of interface fields and currents makes all $Y_{is'}=0$. It is not necessary for field or current symmetry with respect to the x-axis to hold in order to assure that the left-hand side of (66) is zero. The homogeneous set of equations in (63) for the q_{is} coefficients requires

$$\det(S_x) = 0 \tag{67}$$

for a solution to exist. Equation (67) is the characteristic dispersion equation in γ to be solved. In general, (67) produces an infinite set of γ_i eigenvalues, each a complex number $\gamma_i = \alpha_i + j\beta_i$. α_i is the attenuation constant and β_i the phase propagation constant. Phase velocity of the ith eigenvalue is given by $v_{pi} = \omega/\beta_i$ where ω is the radian frequency. Electromagnetic tangential fields can be obtained from (41) in any layer by inserting the calculated γ_i. Examination of (16) shows that it provides the solution for the transverse-field components of either the normal mode solution or the total field solution. For $\tilde{E}_y^m$ and $\tilde{H}_y^m$ components one sees that

$$\begin{bmatrix} \tilde{E}_y^m \\ \tilde{H}_y^m \end{bmatrix} = \begin{bmatrix} a_2^T \\ a_5^T \end{bmatrix} \Lambda^m \tag{68}$$

gives the total field solution result where a_i^T are transposes of the vectors discussed in (45), and (17) is used.

IV. Closed Guided-Wave Structure

Again, reference to Fig. 1 for the abstract representation of the closed waveguide structure to be studied is done as in the previous section. Here, though, there is no need to carry out a limiting process on the $(n+1)$th layer thickness. The $(n+1)$th boundary is retained as a perfectly conducting electric wall (or ground plane). Recall in Section III that (41) was specialized to the situation where the $y=0$ boundary was pulled through n layers and then pushed infinitesimally across the nth interface into the $(n+1)$th region. Here, the mapping is extended to cover the entire $(n+1)$th medium and arrive at the $(n+1)$th boundary. The equation analogous to (43) for the present case is

$$\Lambda^{n+1}(h_{n+1}) = F^{n+1,n+1}(h_{n+1})\Lambda^1(0) + \sum_{k=1}^{n} F^{n+1,n+1-k}(h_{n+1})[0 \quad 0 \quad -\tilde{J}_{sz}^k \quad \tilde{J}_{sx}^k]^T. \tag{69}$$

Equation (69) was found by evaluating (41) at $y'_{n+1} = h_{n+1}$. BC's on the $(n+1)$th boundary impose $\Lambda_1^{n+1}(h_{n+1}) = \Lambda_2^{n+1}(h_{n+1}) = 0$. Putting all the relevant BC's into (69) gives

$$\begin{bmatrix} 0 \\ 0 \\ \tilde{H}_x(h_{n+1}) \\ \tilde{H}_z(h_{n+1}) \end{bmatrix} = F^{n+1,n+1} \begin{bmatrix} 0 \\ 0 \\ \tilde{H}_x^1(0) \\ \tilde{H}_z^1(0) \end{bmatrix} + \sum_{k=1}^{n} F^{n+1,n+1-k} \begin{bmatrix} 0 \\ 0 \\ -\tilde{J}_{sz}^k \\ \tilde{J}_{sx}^k \end{bmatrix} \quad (70)$$

as the equation to be solved for the two nonzero components of $\Lambda^1(0)$ which allows $\Lambda^m(y'_m)$ in (41) to be calculated. Recognizing that only the first two rows of the vectors on either side of the equality sign in (70) are necessary for $\Lambda^1(0)$ component determination, the resulting two linear inhomogeneous equations are solved to yield

$$\Lambda_{i+2}^1(0) = \tilde{H}_i^1(0) = \sum_{j=1}^{2} \sum_{k=1}^{n} I_{ij}^k \tilde{J}_{sj}^k, \qquad i,j = 1,2(x,z) \quad (71)$$

$$I_{ij}^k = (-1)^{i+j} \frac{1}{D_c} \left[F_{2,5-i}^{n+1,n+1} F_{1,5-j}^{n+1,n+1-k} - F_{1,5-i}^{n+1,n+1} F_{2,5-j}^{n+1,n+1-k} \right] \quad (72)$$

$$D_c = \det \begin{pmatrix} F_{13}^{n+1,n+1} & F_{14}^{n+1,n+1} \\ F_{23}^{n+1,n+1} & F_{24}^{n+1,n+1} \end{pmatrix}. \quad (73)$$

$\Lambda^m(y'_m)$ is now known in terms of the interface surface source current components by placing (71) into (41). After carrying out a substantial amount of bookkeeping relating all the interface electric-field and surface current components based on the resulting (41), a Green's function dyadic identical in form to (56) occurs. Submatrices S^{mp} are now defined differently

$$S_{ij}^{mp} = T_{ij}^{mp} - (-1)^j F_{1,5-j}^{m,m-p} \quad (74)$$

$$T_{ij}^{mp} = F_{i3}^{m,m} I_{1j}^p + F_{i4}^{m,m} I_{2j}^p. \quad (75)$$

Complex propagation constant eigenvalues γ_i are ascertained following the same steps discussed between (59) and (67). Tangential and transverse electromagnetic fields are again found, respectively, applying (41) and (68).

V. Conclusion

The great value of the matrix method covered in this paper is that it permits a systematic approach for solving most planar guided-wave problems. Methodology is so general as to afford solutions to the most complex anisotropic layered problems. However, many simpler problems are just as readily solved by the method. Computer program construction based on the method should allow workers in the microwave and millimeter-wave community to easily deal with structures having a few layers to hundreds of layers, but use a technique which avoids the use of mesh sizes and shapes as employed, for example, by finite-difference and finite-element numerical approaches. The matrix method was formulated in the Fourier-transformed domain (or spectral domain) because of the advantage of converting integral expressions or integral equations into algebraic and differential equations. Although the approach in the paper was differential, the reader should be aware that electromagnetic-field information and current-distribution information must still be obtained by converting from the Fourier domain to the real or direct space domain by inverse Fourier transformations. Propagation constant γ information does not require such an inverse transformation (since it is an eigenvalue).

As presented here, the formulation yields, in principle, exact field and propagation constant solutions provided an unlimited amount of basis functions in complete sets are employed and sufficient computer core and time are available. Depending on the specific layered problem attacked, a finite number of basis functions will be required and the problem solvable within an acceptable level of accuracy or approximation. The starting point of the method is the specification or determination of constitutive tensor $\hat{M}_i$ characterizing each layer i. $\hat{M}_i$ should be determined from the physical microscopic or macroscopic properties of each layer material under consideration. Since $\hat{M}_i$ is a macroscopic tensor, it is essential that all microscopic phenomena leading to electromagnetic-field interactions be representable ultimately at the macroscopic level. This conversion from microscopic to macroscopic representation is required for all layers if the formulation technique is to work.

For example, in a relatively thick semiconductor layer where the bulk $\hat{M}_i$ can be used, $\hat{M}_i$ would arise from various microscopic transport effects, such as impurity coulomb scattering, alloy scattering, carrier-carrier scattering via coulomb and exchange interactions, and intravalley and intervalley scattering. Converting the above semiconductor transport effects from the microscopic to the macroscopic level is well understood and regularly done. For layers which are narrow and lead to separation and quantization of carrier energy levels in the transverse or y-direction, this two-dimensional effect may play a noticable role in altering the scattering behavior through change of carrier quantum mechanical wavefunctions Ψ_c. If the layer walls are considered impenetrable to Ψ_c, then size effect, energy level splitting on the order of single particle $\epsilon_n = (n\hbar\pi/h_n)^2/2m^*$ [14] can be expected where $\hbar$ is Planck's constant and m^* the carrier effective mass. In a general layered problem, the approximation of impenetrable walls may be unsuitable, in which case leaky interface walls would exist requiring a simultaneous solution of Ψ_c in all layers in the structure. Such a multilayer determination of Ψ_c is not unlike the macroscopic electromagnetic problem treated in this paper. Nevertheless, however Ψ_c is obtained (including or not including many body effects), it must be utilized to go from microscopic considerations to the macroscopic determination of $\hat{M}_i$. The layer size effect discussed here is also related to similar two-dimensional quantum mechanical phenomena, namely quantum mechanical well creation and transport effects in metal–insulator–semiconductor (MIS) structures and devices [15].

ACKNOWLEDGMENT

The author thanks Drs. S. Teitler and P. J. Lin-Chung for discussions on the use matrix techniques in transmission/reflection work involving layered materials which they are investigating.

REFERENCES

[1] G. A. Prinz and J. J. Krebs, "Molecular beam epitaxial growth of single-crystal Fe films on GaAs," *Appl. Phys. Letts.*, vol. 39, no. 5, pp. 397–399, Sept. 1981.
[2] G. A. Prinz, G. T. Rado, and J. J. Krebs, "Magnetic properties of single-crystal {110} iron films grown on GaAs by molecular beam epitaxy," in *Proc. 27th Ann. Confer. Magn. Magn. Mater., J. Appl. Phys.*, vol. 53, no. 3, part II, Mar. 1982, pp. 2087–2091.
[3] A. M. A. El-Sherbiny, "Hybrid mode analysis of microstrip lines on anisotropic substrates," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 1261–1266, Dec. 1981.
[4] A. M. A. El-Sherbiny, "Millimeter-wave performance of shielded slot-lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 750–756, May 1982.
[5] O. Schwelb, "Network representation and transverse resonance for layered anisotropic dielectric waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 899–905, June 1982.
[6] C. M. Krowne, "Green's function in the spectral domain for biaxial anisotropic planar dielectric structures," in *IEEE Antennas Propagat. Symp. Dig.*, (Boston, MA), June 25–29, 1984, pp. 693–696. Also in *IEEE Trans. Antennas. Propagat.*, vol. AP-32, Dec. 1984.
[7] "28-Year Index," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, part II, pp. 1–207, June 1981. For example, see p. II-83 for papers on anisotropic media, p. II-110 for papers on electromagnetic propagation in anisotropic media and in magnetic media, p. II-111 for papers on electromagnetic propagation in nonhomogeneous media, and p. II-112 for papers on electromagnetic propagation in semiconducting media.
[8] R. C. Jones, "A new calculus for the treatment of optical systems, I. Description and discussion of the calculus," *J. Opt. Soc. Am.*, vol. 31, pp. 488–493, July 1941.
[9] M. Born and E. Wolf, *Principles of Optics*. New York: Pergamon, 1964, p. 808.
[10] J. A. Kong, *Theory of Electromagnetic Waves*. New York: Wiley, 1975, p. 339.
[11] S. Teitler and B. W. Henvis, "Refraction in stratified, anisotropic media," *J. Opt. Soc. Am.*, vol. 60, pp. 830–834, June 1970.
[12] D. W. Berreman and T. J. Scheffer, "Order versus temperature in cholesteric liquid crystals from reflectance spectra," *Phys. Rev.*, vol. A5, pp. 1397–1403, Mar. 1972.
[13] D. W. Berreman, "Optics in stratified and anisotropic media: 4×4-matrix formulation," *J. Opt. Soc. Am.*, vol. 62, pp. 502–510, Apr. 1972.
[14] L. I. Schiff, *Quantum Mechanics*. New York: McGraw-Hill, 1968, p. 544. Also see S. Borowitz, *Fundamentals of Quantum Mechanics*. New York: Benjamin, 1964, p. 401.
[15] C. M. Krowne, "Electron power loss in the (100) n channel of a Si metal-oxide-semiconductor field-effect transistor. Parts I and II, intrasubband and intersubband phonon scattering," *J. Appl. Phys.*, vol. 54, no. 5, pp. 2441–2467, May 1983.

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